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Coherent and Squeezed States for Light in Homogeneous Conducting Linear Media by an Invariant Operator Method

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With the choice of Coulomb gauge, we investigated coherent state and squeezed state of the light propagating through homogeneious conducting linear media with no charge density using quantum results of the LR invariant operator method. We described coherent and squeezed properties of electric and magnetic fields. The fields in coherent and squeezed states are decayed exponentially with time due to the conductivity of the media. We studied probability density of the coherent wave packet and the highly squeezed wave packets. The uncertainty relation between the two orthogonal phase amplitudes, $\hat{a}_1$ and $\hat{a}_2$, in coherent state is same as the uncertainty relation in vacuum number state. The envelope of the relative noise in coherent state alternately become large and small with time and position. The uncertainty relation between canonical variables are varied depending on the value of conductivity σ in squeezed state, but not lowered below $\hbar/2$ which is quantum-mechanically acceptable minimum uncertainty.

KEY WORDS: dissipative light; LR invariant operator method; coherent state; squeezed state.
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1. INTRODUCTION

Glauber proposed standard coherent state for the harmonic oscillator, which is the archetype for most kind of coherent states (Glauber, 1968). The coherent state can be created from the ground state by a displacement operator and can be expanded in terms of the eigenstates of the Hamiltonian system. Coherent state is near to the classical wave as far as quantum mechanics permits (Scully and Zubairy, 1997; Walls and Milburn, 1994), since this state represents well-defined amplitude and phase unlike photon number state. Theoretically, this state can be established by laser light that oscillates with sufficiently large amplitude. The coherent state provides time-dependent behavior of electromagnetic waves that has the form of typical harmonic oscillator (Vogel and Welsch, 1994). Although the coherent state lacks the orthogonal property, they have been used as a set of basis to describe the

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electromagnetic fields, since the state of the fields can be represented uniquely in terms of coherent state which forms a complete set.

The squeezed states as well as coherent states are worth in discussion of the Heigenberg minimal uncertainty relation in quantum optics. The idea of squeezing is important in the realization of reducing noise. The squeeze of the variance of either q or p below the value of coherent state can be accomplished at the expense of enhanced variance in the other quadrature so as to satisfy the requirements of the uncertainty principle.

In ordinary simple harmonic oscillator, the Schrödinger equation in q-space can be solved by separating canonical variable $\hat{q}$ and time t. However, in case of time-dependent harmonic oscillator such as damped harmonic oscillator, the separation of these two variables cannot be performed straightforwardly, since they are entangled together. For this reason, several techniques have been devised in order to quantize time-dependent harmonic oscillator, i.e., LR(Lewis–Riesenfeld) invariant operator method (Choi, 2003a; Choi and Gweon, 2002; Ji and Kim, 1995; Lewis, 1967, 1968; Lewis and Riesenfeld, 1969; Yeon and Kim, 1994), unitary transformation method (Brown, 1991; Li *et al.*, 1994; Choi, 2003b; Zhang *et al.*, 2002), canonical transformation method (Pedrosa, 1987; Um *et al.*, 1998; Yeon *et al.*, 1997, 2001), and propagator method (Gweon and Choi, 2003; Yeon *et al.*, 1993; Um *et al.*, 2000, 2002). Note that all of these methods give the same quantum results.

In homogenous conducting linear media, fields and current satisfy the relations

$$D = \epsilon E, \quad H = B/\mu, \quad J = \sigma E, \tag{1}$$

where ϵ, μ and σ are the electric permittivity, magnetic permeability and conductivity of the media respectively. In the previous paper (Choi, 2003a), we quantized light satisfying these conditions using LR invariant operator method and has been shown that the quantum solution derived from this method for the dissipative light is same as that of damped harmonic oscillator and its energy expectation values in both classical and quantum-mechanical view point are decreased continuously and exponentially as time goes by. In this paper, taking advantage of the quantum results in Ref. (Choi, 2003a), we will investigate various properties of light in coherent state and squeezed states such as expectation values and dispersions of several physical quantities and uncertainty relations between canonical quantum variables.

2. LR INVARIANT OPERATOR FOR THE DISSIPATIVE FIELDS

In previous papers, we obtained the quantum solution of the propagating light in homogeneous conducting linear media (Choi, 2003a) and described the corresponding electromagnetic fields (Choi, 2003c) in the spirit of LR invariant operator method. The choice of the Coulomb gauge in homogeneous linear media that has no charge source enabled us to express the electromagnetic field in terms of only vector potential rather than scalar potential. The variables position and time in vector potential can be separated as

$$\hat{A}(x,t) = u(x)\hat{q}(t). \tag{2}$$

In one dimension, $u(x)$ and $\hat{q}(t)$ satisfies the following differential eq. (Choi, 2003b; Louisell, 1973)

$$\frac{\partial^2 u(x)}{\partial x^2} + \frac{\omega^2}{c^2}u(x) = 0, \tag{3}$$

$$\frac{\partial^2 \hat{q}(t)}{\partial t^2} + \frac{\sigma}{\epsilon}\frac{\partial \hat{q}(t)}{\partial t} + \omega^2\hat{q}(t) = 0, \tag{4}$$

where ω is the natural frequency and $c \equiv 1/\sqrt{\epsilon\mu}$ is the velocity of the light in media. If we consider the propagating wave under periodic boundary conditions, $u(x)$ is given by (Choi, 2003c)

$$u(x) = \frac{1}{\sqrt{V}}\exp(\pm ikx), \tag{5}$$

where V is the volume of the cube and $k = \omega/c$ is the wave number. Since Eq. (4) is same as damped wave equation, the Hamiltonian satisfying Schrödinger equation is expressed in terms of $\hat{q}$ (Choi, 2003a):

$$H(\hat{q},\hat{p},t) = \exp\left(-\frac{\sigma}{\epsilon}t\right)\frac{\hat{p}^2}{2\epsilon} + \frac{1}{2}\exp\left(\frac{\sigma}{\epsilon}t\right)\epsilon\omega^2\hat{q}^2, \tag{6}$$

where $\hat{p} = -i\hbar(\partial/\partial\hat{q})$.

From the following relation for LR invariant operator $\hat{I}$

$$\frac{d\hat{I}}{dt} = \frac{\partial\hat{I}}{\partial t} + \frac{1}{i\hbar}[\hat{I},\hat{H}(\hat{q},\hat{p},t)] = 0, \tag{7}$$

we found that

$$\hat{I} = \frac{1}{2}\hbar\Omega\left(\hat{X}_1^2 + \hat{X}_2^2\right), \tag{8}$$

where Ω is given by $\Omega = \sqrt{\omega^2 - \sigma^2/(4\epsilon^2)}$ and

$$\hat{X}_1 = \sqrt{\frac{\epsilon\Omega}{\hbar}}\exp\left(\frac{\sigma}{2\epsilon}t\right)\hat{q}, \tag{9}$$

$$\hat{X}_2 = \sqrt{\frac{1}{\hbar\epsilon\Omega}}\left[\frac{\sigma}{2}\exp\left(\frac{\sigma}{2\epsilon}t\right)\hat{q} + \exp\left(-\frac{\sigma}{2\epsilon}t\right)\hat{p}\right]. \tag{10}$$

To discuss the photon coherent state, let us introduce the annihilation and creation operators (Choi, 2003a; Choi and Zhang, 2002):

$$\hat{a}(t) = \frac{1}{\sqrt{2}}(\hat{X}_1 + i\hat{X}_2), \tag{11}$$

$$\hat{a}^\dagger(t) = \frac{1}{\sqrt{2}}(\hat{X}_1 - i\hat{X}_2). \tag{12}$$

Then, Eq. (8) can be written as

$$\hat{I} = \hbar\Omega\left(\hat{a}^\dagger(t)\hat{a}(t) + \frac{1}{2}\right). \tag{13}$$

Since $[\hat{X}_1, \hat{X}_2] = i$, the ladder operators satisfy the Boson commutation relation

$$[\hat{a}, \hat{a}^\dagger] = 1. \tag{14}$$

We can easily check that the direct differentiation of $\hat{a}(t)$ and $\hat{a}^\dagger(t)$ with respect to time satisfy

$$\frac{d\hat{a}(t)}{dt} = -i\Omega\hat{a}(t), \tag{15}$$

$$\frac{d\hat{a}^\dagger(t)}{dt} = i\Omega\hat{a}^\dagger(t), \tag{16}$$

whose solutions are given by

$$\hat{a}(t) = \hat{a}(0)e^{-i\Omega t}, \tag{17}$$

$$\hat{a}^\dagger(t) = \hat{a}^\dagger(0)e^{i\Omega t}. \tag{18}$$

We denote the eigenstates of the LR invariant operator by $|n\rangle$:

$$\hat{I}|n\rangle = \lambda_n|n\rangle. \tag{19}$$

Because both $\hat{a}|n\rangle$ and $\hat{a}^\dagger|n\rangle$ are the eigenstates of number operator $N(= \hat{a}^\dagger\hat{a})$ with the corresponding eigenvalue $(n-1)$ and $(n+1)$, respectively, we can confirm that the following conventional relations hold

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle, \tag{20}$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle. \tag{21}$$

From $\hat{a}|0\rangle = 0$, we can easily derive vacuum state of the LR invariant operator. By acting $\hat{a}^\dagger$ to the vacuum state n times,

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}|0\rangle, \tag{22}$$

the nth eigenstate can also be obtained, whose q-space representation is given by

$$\langle q|n\rangle = \frac{1}{\sqrt{2^n n!}}\left(\frac{\epsilon\Omega}{\pi\hbar}\right)^{1/4} H_n\left[\sqrt{\frac{\epsilon\Omega}{\hbar}}\exp\left(\frac{\sigma}{2\epsilon}t\right)q\right]$$
$$\times \exp\left\{\frac{\sigma}{4\epsilon}t - \frac{1}{2\hbar}\exp\left(\frac{\sigma}{\epsilon}t\right)\left(\epsilon\Omega + \frac{i\sigma}{2}\right)q^2\right\}. \tag{23}$$

In the above equation, H_n is nth order Hermite polynomial. The time-dependent full wave functions $\langle q|\psi_n\rangle$ that satisfying the Schrödinger equation are same as the eigenstates of the LR invariant operator, except for a time-dependent phase factor (Lewis and Riesenfeld, 1969; Choi, 2003a):

$$\langle q|\psi_n\rangle = \langle q|n\rangle \exp\left[-i\left(n+\frac{1}{2}\right)\Omega t\right]. \tag{24}$$

If we consider Eq. (24), Eqs. (20) and (21) can be rewritten in terms of $|\psi_n\rangle$ as

$$\hat{a}|\psi_n\rangle = \sqrt{n}\, e^{-i\Omega t}|\psi_{n-1}\rangle, \tag{25}$$

$$\hat{a}^\dagger|\psi_n\rangle = \sqrt{n+1}\, e^{i\Omega t}|\psi_{n+1}\rangle. \tag{26}$$

3. COHERENT STATE OF DISSIPATIVE LIGHT

Let us denote the eigenvalue and eigenstate of annihilation operator $\hat{a}$ as α and $|\alpha\rangle$:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle. \tag{27}$$

It is well known that the coherent state can be expanded in terms of $|n\rangle$ as (Choi, 2004; Dantas *et al.*, 1992; Louisell, 1973)

$$|\alpha\rangle = \sum_n c_n(\alpha)|n\rangle, \tag{28}$$

where the coefficients $c_n(\alpha)$ is the transformation function between number state and coherent state. The transformation function is given by

$$c_n(\alpha) = \exp\left(-\frac{1}{2}|\alpha|^2\right)\frac{\alpha^n}{\sqrt{n!}}. \tag{29}$$

The probability that the coherent state resides in nth state of harmonic oscillator is same as $|c_n(\alpha)|^2$:

$$|c_n(\alpha)|^2 = \frac{\langle n\rangle_\alpha^n \exp(-\langle n\rangle_\alpha)}{n!}, \tag{30}$$

where $\langle n\rangle_\alpha$ is the expectation value of photon number n with respect to $|\alpha\rangle$. In this paper, we will abbreviate $\langle\alpha|\cdots|\alpha\rangle$ to $\langle\cdots\rangle_\alpha$ for convenient. By inserting

Eq. (22) into Eq. (28), we can confirm that the coherent state can also be obtained by operating displacement operator to vacuum state:

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle, \tag{31}$$

where the displacement operator $\hat{D}(\alpha)$ is given by

$$\hat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a}). \tag{32}$$

The displacement operator has usual property that transforms $\hat{a}$ and $\hat{a}^\dagger$ as, respectively

$$\hat{D}^\dagger(\alpha)\,\hat{a}\,\hat{D}(\alpha) = \hat{a} + \alpha, \tag{33}$$

$$\hat{D}^\dagger(\alpha)\,\hat{a}^\dagger\,\hat{D}(\alpha) = \hat{a}^\dagger + \alpha^*. \tag{34}$$

By substitution of Eqs. (23) and (29) into Eq. (28), we can easily obtain the coherent state in q-space as

$$\langle q|\alpha\rangle = \left(\frac{\epsilon\Omega}{\pi\hbar}\right)^{1/4} \exp\left\{\frac{1}{\hbar}\exp\left(\frac{\sigma}{2\epsilon}t\right)\left[\sqrt{2\hbar\epsilon\Omega}\alpha q - \frac{1}{2}\left(\epsilon\Omega + i\frac{\sigma}{2}\right)\right.\right.$$
$$\left.\left.\times\exp\left(\frac{\sigma}{2\epsilon}t\right)q^2\right] + \frac{\sigma}{4\epsilon}t - \frac{1}{2}\alpha^2 - \frac{1}{2}|\alpha|^2\right\}. \tag{35}$$

Performing the similar procedure in p-space, we found that

$$\langle p|\alpha\rangle = \left(\frac{\Omega}{\pi\hbar\epsilon}\right)^{1/4} \frac{1}{\sqrt{\Omega + i\sigma/(2\epsilon)}} \exp\left\{-\frac{\sigma}{4\epsilon}t - \frac{1}{\hbar\epsilon[\Omega + i\sigma/(2\epsilon)]}\right.$$
$$\times\exp\left(-\frac{\sigma}{2\epsilon}t\right)\left[i\alpha\sqrt{2\hbar\epsilon\Omega}p + \frac{1}{2}\exp\left(-\frac{\sigma}{2\epsilon}t\right)p^2\right]$$
$$\left. + \frac{1}{2}\frac{\Omega - i\sigma/(2\epsilon)}{\Omega + i\sigma/(2\epsilon)}\alpha^2 - \frac{1}{2}|\alpha|^2\right\}. \tag{36}$$

If we set $\sigma = 0$, Eqs. (35) and (36), exactly reduces to that of ordinary simple harmonic oscillator which is minimum wave packet in the form of Gaussian. The formulas Eqs. (11) and (12) with Eqs. (9) and (10), can be inverted to yield

$$\hat{q} = \sqrt{\frac{\hbar}{2\epsilon\Omega}}\exp\left(-\frac{\sigma}{2\epsilon}t\right)\left[\hat{a}^\dagger(t) + \hat{a}(t)\right], \tag{37}$$

$$\hat{p} = i\sqrt{\frac{\epsilon\Omega\hbar}{2}}\exp\left(\frac{\sigma}{2\epsilon}t\right)\left\{\left(1 + i\frac{\sigma}{2\Omega\epsilon}\right)\hat{a}^\dagger(t) - \left(1 - i\frac{\sigma}{2\Omega\epsilon}\right)\hat{a}(t)\right\}. \tag{38}$$

The expectation value of canonical variables in coherent state can be calculated as

$$\langle\hat{q}\rangle_\alpha = \sqrt{\frac{2\hbar}{\epsilon\Omega}}\exp\left(-\frac{\sigma}{2\epsilon}t\right)\mathrm{Re}\,\alpha, \tag{39}$$

$$\langle \hat{p} \rangle_\alpha = \sqrt{2\epsilon\Omega\hbar}\exp\left(\frac{\sigma}{2\epsilon}t\right)\left(\mathrm{Im}\,\alpha - \frac{\sigma}{2\Omega\epsilon}\mathrm{Re}\,\alpha\right), \tag{40}$$

where Re α and Im α mean the real and imaginary parts of α. Using Eqs. (37) and (38) we can evaluate the variances of $\hat{q}$ and $\hat{p}$ as follows, respectively

$$(\Delta\hat{q})_\alpha = \left[\langle \hat{q}^2 \rangle_\alpha - \langle \hat{q} \rangle_\alpha^2\right]^{1/2} = \sqrt{\frac{\hbar}{2\epsilon\Omega}}\exp\left(-\frac{\sigma}{2\epsilon}t\right), \tag{41}$$

$$(\Delta\hat{p})_\alpha = \left[\langle \hat{p}^2 \rangle_\alpha - \langle \hat{p} \rangle_\alpha^2\right]^{1/2} = \sqrt{\frac{\epsilon\Omega\hbar}{2}\left(1 + \frac{\sigma^2}{4\Omega^2\epsilon^2}\right)}\exp\left(\frac{\sigma}{2\epsilon}t\right). \tag{42}$$

The uncertainty product in coherent state is therefore

$$(\Delta\hat{q})_\alpha(\Delta\hat{p})_\alpha = \frac{\hbar\omega}{2\Omega}. \tag{43}$$

Eq. (43) is exactly same as that of minimum value in number state and we can see that the uncertainty principle hold since $(\Delta\hat{q})_\alpha(\Delta\hat{p})_\alpha$ is always larger than $\hbar/2$. For larger conductivity (and small electric permittivity), the uncertainty product in coherent state is also large. From Eqs. (39) and (40), we can represent the real and imaginary part of α as

$$\mathrm{Re}\,\alpha = \sqrt{\frac{\epsilon\Omega}{2\hbar}}\exp\left(\frac{\sigma}{2\epsilon}t\right)\langle \hat{q} \rangle_\alpha, \tag{44}$$

$$\mathrm{Im}\,\alpha = \frac{1}{\sqrt{2\hbar\epsilon\Omega}}\left[\frac{\sigma}{2}\exp\left(\frac{\sigma}{2\epsilon}t\right)\langle \hat{q} \rangle_\alpha + \exp\left(-\frac{\sigma}{2\epsilon}t\right)\langle \hat{p} \rangle_\alpha\right]. \tag{45}$$

Combining the above two equations give

$$\alpha = \sqrt{\frac{\epsilon\Omega}{2\hbar}}\left(1 + i\frac{\sigma}{2\Omega\epsilon}\right)\exp\left(\frac{\sigma}{2\epsilon}t\right)\langle \hat{q} \rangle_\alpha + \frac{i}{\sqrt{2\hbar\epsilon\Omega}}\exp\left(-\frac{\sigma}{2\epsilon}t\right)\langle \hat{p} \rangle_\alpha. \tag{46}$$

The eigenvalue α is complex number while $\langle \hat{q} \rangle_\alpha$ and $\langle \hat{p} \rangle_\alpha$ are real numbers. The value α doesn't make orthogonal system since it is defined with continuously variable number in real and imaginary axes, i.e.,

$$|\langle \alpha | \alpha' \rangle|^2 = \exp(-|\alpha - \alpha'|^2). \tag{47}$$

Here we see that α approximately satisfies the orthogonal property only when the difference between α and α' is large and degenerate of Poisson distribution is small. However coherent state satisfies the property of completeness:

$$\int |\alpha\rangle\langle\alpha| d^2\alpha = \pi. \tag{48}$$

We can find the transformation function $\langle q' | \alpha \rangle$ by acting conjugate state $\langle q' |$ which corresponds to the eigenvalue q' of $\hat{q}$ to both side of Eq. (27) after inserting Eq. (11)

for $\hat{a}$ as follows (Louisell, 1973)

$$\langle q'|\alpha\rangle = \left(\frac{\epsilon\Omega}{\pi\hbar}\right)^{1/4} \exp\left\{-\frac{1}{2\hbar}\exp\left(\frac{\sigma}{\epsilon}t\right)\left[\epsilon\Omega(q'-\langle\hat{q}\rangle_\alpha)^2 - i\sigma\left(q'\langle\hat{q}\rangle_\alpha - \frac{1}{2}q'^2\right)\right] + i\frac{\langle\hat{p}\rangle_\alpha}{\hbar}q' + \frac{\sigma}{4\epsilon}t + i\delta_{cq}\right\}, \tag{49}$$

where δ_{cq} is an arbitrary real phase. With no loss of generality, let us choose δ_{cq} as

$$\delta_{cq} = 4(\mathrm{Re}\,\alpha)(\mathrm{Im}\,\alpha), \tag{50}$$

then, Eq. (49) become just the same as Eq. (35). We can also find the transformation function $\langle p'|\alpha\rangle$ by acting $\langle p'|$ to Eq. (27) as follows

$$\langle p'|\alpha\rangle = \left(\frac{\Omega}{\pi\hbar\epsilon}\right)^{1/4} \frac{1}{\sqrt{\Omega + i\sigma/(2\epsilon)}} \exp\left\{-\frac{1}{2\hbar\epsilon^2[\Omega^2+\sigma^2/(4\epsilon^2)]} \times \exp\left(-\frac{\sigma}{\epsilon}t\right)\left[\epsilon\Omega(p'-\langle\hat{p}\rangle_\alpha)^2 + i\sigma\left(\langle\hat{p}\rangle_\alpha p' - \frac{1}{2}p'^2\right)\right] - \frac{i}{\hbar}\langle\hat{q}\rangle_\alpha p' - \frac{\sigma}{4\epsilon}t + i\delta_{cp}\right\}, \tag{51}$$

where δ_{cp} is another arbitrary real phase. If we choose

$$\delta_{cp} = \frac{1}{2[\Omega^2+\sigma^2/(4\epsilon^2)]}\left\{2\left(\Omega^2 - \frac{\sigma^2}{4\epsilon^2}\right)(\mathrm{Re}\,\alpha)\ (\mathrm{Im}\,\alpha) - \frac{\Omega\sigma}{\epsilon}\left[(\mathrm{Re}\,\alpha)^2 - (\mathrm{Im}\,\alpha)^2\right]\right\}, \tag{52}$$

then, the expression of Eq. (51) recovers to Eq. (36). The probability densities $|\langle q'|\alpha\rangle|^2$ and $|\langle p'|\alpha\rangle|^2$ in coherent state are given by

$$|\langle q'|\alpha\rangle|^2 = \left(\frac{\epsilon\Omega}{\pi\hbar}\right)^{1/2} \exp\left\{\frac{1}{\hbar}\exp\left(\frac{\sigma}{2\epsilon}t\right)\left[2\sqrt{2\hbar\epsilon\Omega}\,(\mathrm{Re}\,\alpha)q' - \epsilon\Omega \times \exp\left(\frac{\sigma}{2\epsilon}t\right)q'^2\right] + \frac{\sigma}{2\epsilon}t - (\mathrm{Re}\,\alpha)^2 + (\mathrm{Im}\,\alpha)^2 - |\alpha|^2\right\}, \tag{53}$$

$$|\langle p'|\alpha\rangle|^2 = \left(\frac{\Omega}{\pi\hbar\epsilon}\right)^{1/2} \frac{1}{\sqrt{\Omega^2+\sigma^2/(4\epsilon^2)}} \exp\left\{-\frac{\sigma}{2\epsilon}t - \frac{1}{\hbar\epsilon[\Omega^2+\sigma^2/(4\epsilon^2)]} \times \exp\left(-\frac{\sigma}{2\epsilon}t\right)\left[\Omega\exp\left(-\frac{\sigma}{2\epsilon}t\right)p'^2 + \sigma\sqrt{\frac{2\hbar\Omega}{\epsilon}}(\mathrm{Re}\,\alpha)p'\right]\right.$$

$$+\frac{\Omega^2-\sigma^2/(4\epsilon^2)}{\Omega^2+\sigma^2/(4\epsilon^2)}[(\mathrm{Re}\,\alpha)^2-(\mathrm{Im}\,\alpha)^2]+\frac{2\Omega\sigma}{\epsilon[\Omega^2+\sigma^2/(4\epsilon^2)]}$$
$$\times(\mathrm{Re}\,\alpha)(\mathrm{Im}\,\alpha)-|\alpha|^2\Bigg\}. \tag{54}$$

Figure 1 represents the graph of probability densities given in Eqs. (53) and (54). From these figures, we can confirm that the probability densities oscillate back and forth with time about $q'=0$ and $p'=0$, respectively. This behavior is very similar to the motion of classical oscillator. The amplitude of oscillation decreases with time in q-space while increases in p-space.

Let us consider a radiation field with photon annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$ and suppose that the state of the electromagnetic field is described by a coherent state density operator $\hat{\rho}$ that given by

$$\hat{\rho}=|\alpha\rangle\langle\alpha|. \tag{55}$$

From the above definition of $\hat{\rho}$, it follows that the density operator is Hermitian $\hat{\rho}^\dagger=\hat{\rho}$. If we want to take a piece of the system's information in coherent state, we must evaluate the expectation value of the corresponding operator $f(\hat{a},\hat{a}^\dagger)$,

$$\langle f(\hat{a},\hat{a}^\dagger)\rangle_\alpha=\mathrm{Tr}[\hat{\rho}f(\hat{a},\hat{a}^\dagger)]. \tag{56}$$

The conservation of probability density is represented by:

$$\mathrm{Tr}(\hat{\rho})=1. \tag{57}$$

Up to the present, we managed the coherent state of single mode. But, note that the total coherent light can be represented by production of individual modes l as

$$|\{\alpha_l\}\rangle=\prod|\alpha_l\rangle_l. \tag{58}$$

4. COHERENT PROPERTIES OF THE FIELDS

When we recall Eqs. (5) and (37), the vector potential Eq. (2) for the field propagating in x direction is described as

$$\hat{A}(x,t)=\sqrt{\frac{\hbar}{2\epsilon V}}\exp\Big(-\frac{\sigma}{2\epsilon}t\Big)\frac{1}{\sqrt{\Omega}}\big[\hat{a}(0)e^{i(kx-\Omega t)}+\hat{a}^\dagger(0)e^{-i(kx-\Omega t)}\big]. \tag{59}$$

Since the scalar potential is zero (Nayfeh and Brussel, 1985) in media that has no charge density under the choice of Coulomb gauge, the operator representation of electric and magnetic field can be obtained by expansion of only vector potential, Eq. (59),

$$\hat{E}(x,t)=\sqrt{\frac{\hbar}{2\epsilon V}}\exp\Big(-\frac{\sigma}{2\epsilon}t\Big)\frac{1}{\sqrt{\Omega}}\Big[\Big(\frac{\sigma}{2\epsilon}+i\Omega\Big)\hat{a}(0)e^{i(kx-\Omega t)}$$

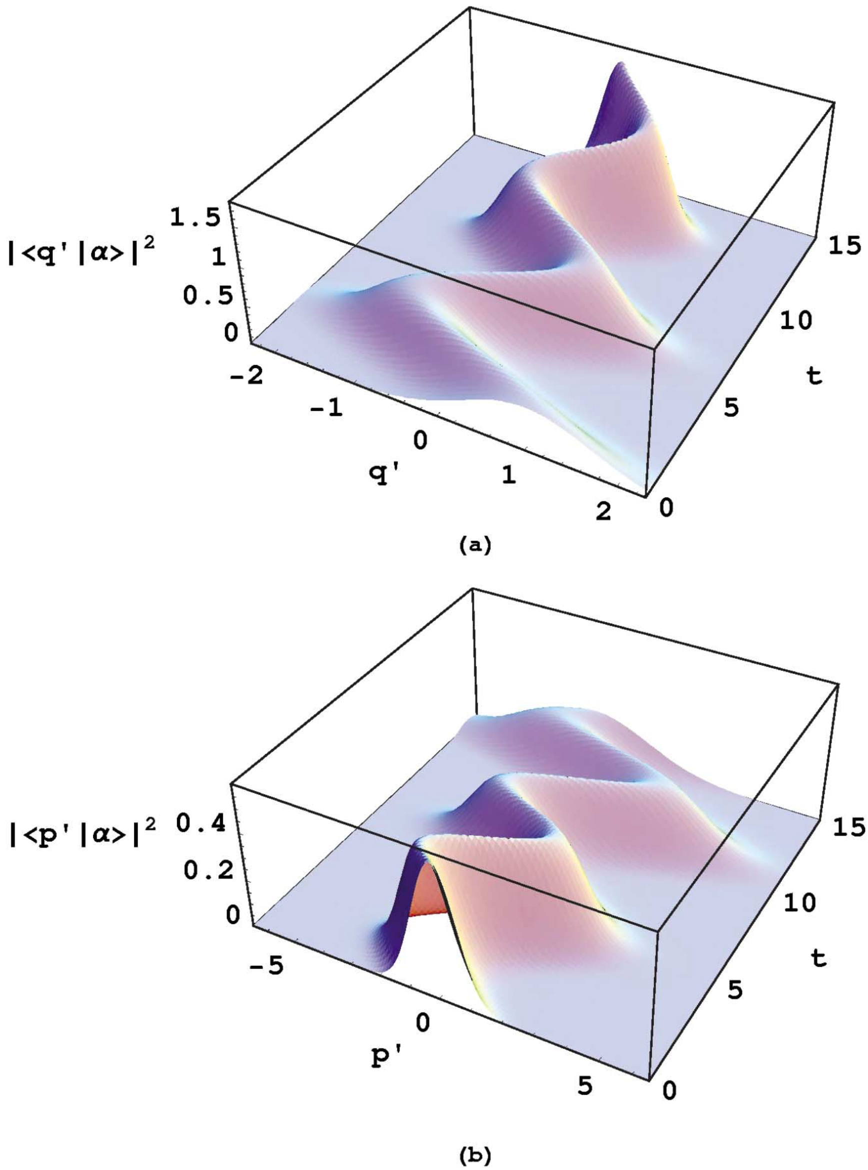


Fig. 1. Probability densities $|\langle q'|\alpha\rangle|^2$ (a) and $|\langle p'|\alpha\rangle|^2$ (b) in coherent state as functions of canonical variable and time. We used $\alpha = |\alpha|e^{-i(\Omega t+\phi)}$ with $|\alpha| = 0.7$ and $\phi = 0$, $\hbar = 1$, $\epsilon = 1$, $\sigma = 0.14$, and $\omega = 1$.

$$+ \left(\frac{\sigma}{2\epsilon} - i\Omega\right) \hat{a}^\dagger(0) e^{-i(kx-\Omega t)}\Big], \tag{60}$$

$$\hat{B}(x,t) = i\sqrt{\frac{\hbar}{2\epsilon V}} \exp\left(-\frac{\sigma}{2\epsilon}t\right) \frac{k}{\sqrt{\Omega}}\Big[\hat{a}(0)e^{i(kx-\Omega t)} - \hat{a}^\dagger(0)e^{-i(kx-\Omega t)}\Big]. \tag{61}$$

From Eqs. (60) and (61), we can confirm that the fields decay exponentially, and eventually disappear on account of conductive property of media. The dissipated field energy become the source of the heat produced in raising the temperature in the media (Griffiths, 1989; Reitz *et al.*, 1993).

Let us separate ladder operators into real and imaginary part as

$$\hat{a} = \hat{a}_1 + i\hat{a}_2, \quad \hat{a}^\dagger = \hat{a}_1 - i\hat{a}_2. \tag{62}$$

The real number $\hat{a}_1$ and $\hat{a}_2$ are called orthogonal phase amplitudes while $\hat{a}$ and $\hat{a}^\dagger$ are complex amplitudes. Then Eqs. (60) and (61) can be reexpressed as

$$\hat{E}(x,t) = \sqrt{\frac{2\hbar}{\epsilon V\Omega}} \exp\left(-\frac{\sigma}{2\epsilon}t\right) \left\{\left[\frac{\sigma}{2\epsilon}\hat{a}_1(0) - \Omega\hat{a}_2(0)\right]\cos(kx - \Omega t)\right.$$
$$\left. - \left[\frac{\sigma}{2\epsilon}\hat{a}_2(0) + \Omega\hat{a}_1(0)\right]\sin(kx-\Omega t)\right\}, \tag{63}$$

$$\hat{B}(x,t) = -k\sqrt{\frac{2\hbar}{\epsilon V\Omega}} \exp\left(-\frac{\sigma}{2\epsilon}t\right)[\hat{a}_1(0)\sin(kx-\Omega t) + \hat{a}_2(0)\cos(kx-\Omega t)]. \tag{64}$$

Consideration of Eq. (14) and Eq. (62) enables us to write $[\hat{a}_1, \hat{a}_2] = i/2$. Then, we can derive the following uncertainty relation in number state

$$(\Delta\hat{a}_1)_n(\Delta\hat{a}_2)_n = \frac{1}{2}\left(n + \frac{1}{2}\right), \tag{65}$$

where $(\Delta\hat{a}_i)_n = \sqrt{\langle n|\hat{a}_i^2|n\rangle - \langle n|\hat{a}_i|n\rangle^2}$ for $i = 1, 2$. On the other hand, when we investigate this relation in coherent state, we obtain that

$$(\Delta\hat{a}_1)_\alpha(\Delta\hat{a}_2)_\alpha = \frac{1}{4}, \tag{66}$$

where $(\Delta\hat{a}_i)_\alpha = \sqrt{\langle\hat{a}_i^2\rangle_\alpha - \langle\hat{a}_i\rangle_\alpha^2}$ for $i = 1, 2$. By comparing Eq. (66) with Eq. (65), we can confirm that the uncertainty relation between the two orthogonal phase amplitudes in coherent state is same as the minimum uncertainty relation in number state.

We can represent complex number α as

$$\alpha(t) = |\alpha(t)|e^{i\theta(t)}, \tag{67}$$

where $|\alpha(t)|$ and $\theta(t)$ are real number which stand for amplitude and phase respectively, which can be obtained from Eq. (46) as

$$|\alpha(t)| = \frac{1}{\sqrt{2\hbar}} \exp\left(\frac{\sigma}{2\epsilon}t\right) \langle\hat{q}\rangle_\alpha \left\{\epsilon\Omega + \frac{1}{\epsilon\Omega}\left[\frac{\sigma}{2} + \exp\left(-\frac{\sigma}{\epsilon}t\right)\frac{\langle\hat{p}\rangle_\alpha}{\langle\hat{q}\rangle_\alpha}\right]^2\right\}^{1/2}, \tag{68}$$

$$\theta(t) = \tan^{-1}\left\{\frac{1}{\epsilon\Omega}\left[\frac{\sigma}{2} + \exp\left(-\frac{\sigma}{\epsilon}t\right)\frac{\langle\hat{p}\rangle_\alpha}{\langle\hat{q}\rangle_\alpha}\right]\right\}. \tag{69}$$

Since $\langle\hat{q}\rangle_\alpha$ and $\langle\hat{p}\rangle_\alpha$ are similar to the classical coordinate and momentum in mechanical system, respectively, they can be derived from Eq. (6) using Hamiltonian dynamics:

$$\langle\hat{q}\rangle_\alpha = q_0 \exp\left(-\frac{\sigma}{2\epsilon}t\right)\cos(\Omega t + \phi), \tag{70}$$

$$\langle\hat{p}\rangle_\alpha = -q_0\epsilon \exp\left(\frac{\sigma}{2\epsilon}t\right)\left[\left(\frac{\sigma}{2\epsilon}\right)\cos(\Omega t + \phi) + \Omega\sin(\Omega t + \phi)\right], \tag{71}$$

where q_0 and ϕ are real amplitude and phase at $t = 0$. Then, we can easily see that Eqs. (68) and (69) are simplified to

$$|\alpha(t)| = |\alpha(0)| = \sqrt{\frac{\epsilon\Omega}{2\hbar}}q_0, \tag{72}$$

$$\theta(t) = -(\Omega t + \phi). \tag{73}$$

If we calculate the mean values of the electric field and its square in coherent state, we have, by Eqs. (27), (60), and (67) with Eqs. (72) and (73),

$$\langle\hat{E}(x,t)\rangle_\alpha = \sqrt{\frac{\hbar}{2\epsilon V\Omega}} \exp\left(-\frac{\sigma}{2\epsilon}t\right)|\alpha(0)|\left[\frac{\sigma}{\epsilon}\cos(kx - \Omega t - \phi) - 2\Omega \times \sin(kx - \Omega t - \phi)\right], \tag{74}$$

$$\langle\hat{E}^2(x,t)\rangle_\alpha = \frac{\hbar}{2\epsilon V\Omega} \exp\left(-\frac{\sigma}{\epsilon}t\right)\left\{|\alpha(0)|^2\left[\frac{\sigma}{\epsilon}\cos(kx - \Omega t - \phi) - 2\Omega \times \sin(kx - \Omega t - \phi)\right]^2 + \left(\frac{\sigma}{2\epsilon}\right)^2 + \Omega^2\right\}, \tag{75}$$

It is also seen that the mean values of the magnetic field and its square in coherent state is, by Eqs. (27), (61) and (67),

$$\langle\hat{B}(x,t)\rangle_\alpha = -k\sqrt{\frac{2\hbar}{\epsilon V\Omega}}|\alpha(0)|\exp\left(-\frac{\sigma}{2\epsilon}t\right)\sin(kx - \Omega t - \phi), \tag{76}$$

$$\langle\hat{B}^2(x,t)\rangle_\alpha = \frac{\hbar k^2}{2\epsilon V\Omega}\exp\left(-\frac{\sigma}{\epsilon}t\right)[4|\alpha(0)|^2\sin^2(kx - \Omega t - \phi) + 1]. \tag{77}$$

From Eqs. (74)–(77), we easily see that the dispersion of $\hat{E}$ and $\hat{B}$ field are

$$[\Delta\hat{E}(x,t)]_\alpha = \sqrt{\langle\hat{E}^2(x,t)\rangle_\alpha - \langle\hat{E}(x,t)\rangle_\alpha^2}$$
$$= \sqrt{\frac{\hbar}{2\epsilon V\Omega}\left[\left(\frac{\sigma}{2\epsilon}\right)^2 + \Omega^2\right]}\exp\left(-\frac{\sigma}{2\epsilon}t\right), \tag{78}$$

$$[\Delta\hat{B}(x,t)]_\alpha = \sqrt{\langle\hat{B}^2(x,t)\rangle_\alpha - \langle\hat{B}(x,t)\rangle_\alpha^2}$$
$$= \sqrt{\frac{\hbar}{2\epsilon V\Omega}}k\exp\left(-\frac{\sigma}{2\epsilon}t\right). \tag{79}$$

Thus, the dispersion for fields also decreases exponentially with time as well as the field strength. The relative noise of the electric field strengths ($\mathrm{RN_E}$) and magnetic field strengths ($\mathrm{RN_B}$) in the coherent state are given by, respectively

$$\mathrm{RN_E} \equiv \left(\frac{[\Delta\hat{E}(x,t)]_\alpha^2}{\langle\hat{E}(x,t)\rangle_\alpha^2}\right)^{1/2}$$
$$= \frac{\sqrt{[\sigma/(2\epsilon)]^2 + \Omega^2}}{|\alpha(0)||(\sigma/\epsilon)\cos(kx - \Omega t - \phi) - 2\Omega\sin(kx - \Omega t - \phi)|}, \tag{80}$$

$$\mathrm{RN_B} \equiv \left(\frac{[\Delta\hat{B}(x,t)]_\alpha^2}{\langle\hat{B}(x,t)\rangle_\alpha^2}\right)^{1/2} = \frac{1}{2|\alpha(0)||\sin(kx - \Omega t - \phi)|}. \tag{81}$$

Figure 2 represents the graph of the relative noise, Eqs. (80) and (81), respectively. From these figures we can see that the envelope of the relative noise alternately become large and small with position as well as with time. Although the field strengths are decrease, the relative noise do not disappear as time goes by. The strength of relative noise depend on the absolute value of α, i.e., for large α it becomes small. The dispersion for the number operator can be evaluated from the relations that $\langle n\rangle_\alpha = |\alpha|^2$ and $\langle n^2\rangle_\alpha = |\alpha|^2 + |\alpha|^4$ as

$$(\Delta n)_\alpha = |\alpha|. \tag{82}$$

Thus, we can see that the absolute value of α, Eq. (68), is same as the dispersion of photon number. The value $|\alpha|^2$ represents the mean value of photon number.

5. SQUEEZED STATE OF DISSIPATIVE LIGHT

We now investigate the properties of the dissipative light in squeezed states. To do this, we introduce the squeeze operator that defined by (Vourdas and

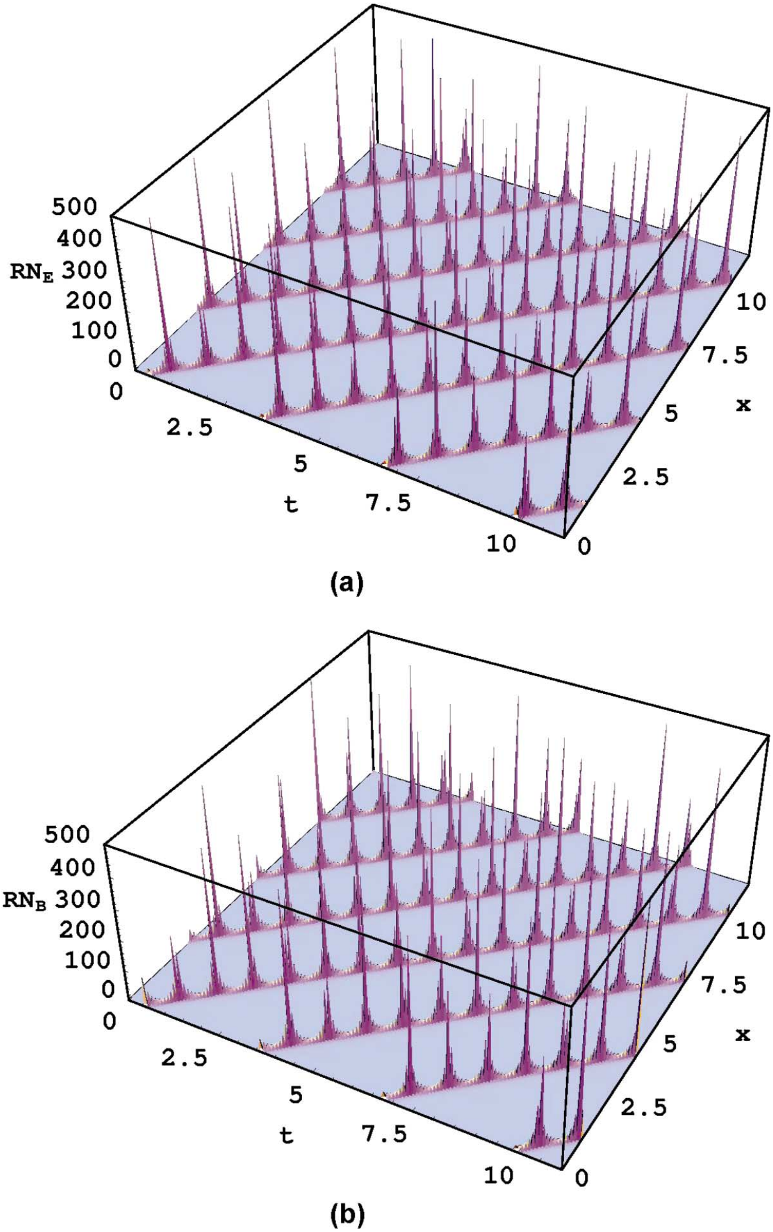


Fig. 2. Relative noise of the electric field (a), and magnetic field (b) strengths given in Eqs. (80) and (81) as functions of position and time. We used $k = 1$, $\hbar = 1$, $\epsilon = 1$, $\sigma = 0.2$ and $\omega = 1$, $|\alpha(0)| = 1$, and $\phi = 0.5$.

Weiner, 1987)

$$\hat{S}(\xi,\lambda) = \exp\left\{-\frac{1}{2}[\xi(\hat{a}^\dagger)^2 - \xi^*\hat{a}^2]\right\}\exp(i\lambda\hat{a}^\dagger\hat{a}), \tag{83}$$

where

$$\xi = |\xi|e^{i\varphi}. \tag{84}$$

Let us denote the transformed operator of $\hat{a}$ and $\hat{a}^\dagger$ by $\hat{S}(\xi,\lambda)$ as $\hat{b}$ and $\hat{b}^\dagger$:

$$\hat{b} = \hat{S}(\xi,\lambda)\,\hat{a}\hat{S}^\dagger(\xi,\lambda), \tag{85}$$

$$\hat{b}^\dagger = \hat{S}(\xi,\lambda)\,\hat{a}^\dagger\hat{S}^\dagger(\xi,\lambda). \tag{86}$$

Then, we found that

$$\hat{b} = \mu\hat{a} + \nu\hat{a}^\dagger, \tag{87}$$

$$\hat{b}^\dagger = \nu^*\hat{a} + \mu^*\hat{a}^\dagger, \tag{88}$$

where

$$\mu = e^{-i\lambda}\cosh|\xi|, \quad \nu = e^{-i(\lambda-\varphi)}\sinh|\xi|. \tag{89}$$

We can easily see that μ and ν satisfies

$$|\mu|^2 - |\nu|^2 = 1, \tag{90}$$

so that $[\hat{b},\hat{b}^\dagger] = 1$. Since the squeezed state $|\beta\rangle$ is the eigenstate of $\hat{b}$, we write eigenvalue equation for $\hat{b}$ as

$$\hat{b}|\beta\rangle = \beta|\beta\rangle. \tag{91}$$

We can find the transformation function $\langle q'|\beta\rangle$ by acting conjugate state $\langle q'|$ which corresponds to the eigenvalue q' of $\hat{q}$ to both side of the above equation after inserting Eq. (87) for $\hat{b}$ to be

$$\langle q'|\beta\rangle = N_q\exp\left\{-\frac{1}{2\hbar}\exp\left(\frac{\sigma}{\epsilon}t\right)\left[\left(\frac{\mu+\nu}{\mu-\nu}\epsilon\Omega + i\frac{\sigma}{2}\right)q'^2 - 2\sqrt{2\hbar\epsilon\Omega}\frac{\mu\alpha+\nu\alpha^*}{\mu-\nu}\exp\left(-\frac{\sigma}{2\epsilon}t\right)q'\right]\right\}, \tag{92}$$

where normalization factor N_q is

$$N_q = \left(\frac{\epsilon\Omega}{\hbar\pi(\mu-\nu)(\mu^*-\nu^*)}\right)^{1/4}\exp\left[-\frac{1}{4}\left(\frac{(\alpha+\alpha^*)^2}{(\mu-\nu)(\mu^*-\nu^*)} - \frac{\sigma}{\epsilon}t\right) + i\delta_{sq}\right], \tag{93}$$

with arbitrary phase δ_{sq}. By using similar way, we also derive the transformation function $\langle p'|\beta\rangle$ as

$$\langle p'|\beta\rangle = N_p \exp\left\{-\frac{1}{2\hbar}\exp\left(-\frac{\sigma}{\epsilon}t\right)\left[\epsilon\Omega(\mu+\nu)+i\frac{\sigma}{2}(\mu-\nu)\right]^{-1}\right.$$
$$\left.\times\left[(\mu-\nu)p'^2+2i(\mu\alpha+\nu\alpha^*)\sqrt{2\hbar\epsilon\Omega}\exp\left(\frac{\sigma}{2\epsilon}t\right)p'\right]\right\}, \quad (94)$$

where

$$N_p = \left(\frac{1}{\hbar\pi\epsilon\Omega\Pi^2}\right)^{1/4}\exp\left\{\frac{1}{4}\left(\frac{[\alpha-\alpha^*-i\sigma(\alpha+\alpha^*)/(2\epsilon\Omega)]^2}{|\Pi|^2}-\frac{\sigma}{\epsilon}t\right)+i\delta_{sp}\right\}, \quad (95)$$

with arbitrary phase δ_{sp} and

$$\Pi = \mu+\nu+\frac{i\sigma}{2\epsilon\Omega}(\mu-\nu). \quad (96)$$

For $\mu = 1$ and $\nu = 0$, the system becomes coherent state and we can show that Eqs. (92) and (94) recovers to Eqs. (49) and (51), respectively, by take advantage of Eq. (46). Figure 3 is the probability densities for the state of highly squeezed in q. We see that the width of the probability density at certain time in q-space is very narrow while that in p-space is much broad so that the uncertainty in q-space is fairly reduced. On the other hand, for the state of highly squeezed in p-space, that depicted in Fig. 4, the uncertainty in p-space is shrunk.

A squeezed state can also be obtained by first acting the squeeze operator $\hat{S}(\xi,\lambda)$ on the vacuum state followed by the displacement operator $\hat{D}(\alpha)$, i.e.,

$$|\beta\rangle = \hat{D}(\alpha)\hat{S}(\xi,\lambda)|0\rangle. \quad (97)$$

The operator expectation values of the squeezed state can be evaluated by making use of the above equation as (Vogel and Welsch, 1994)

$$\langle\beta|\hat{a}|\beta\rangle = \alpha, \quad (98)$$

$$\langle\beta|\hat{a}^\dagger|\beta\rangle = \alpha^*, \quad (99)$$

$$\langle\beta|\hat{a}^2|\beta\rangle = \alpha^2-\mu\nu, \quad (100)$$

$$\langle\beta|(\hat{a}^\dagger)^2|\beta\rangle = (\alpha^*)^2-\mu^*\nu^*, \quad (101)$$

$$\langle\beta|\hat{a}^\dagger\hat{a}|\beta\rangle = |\alpha|^2+|\nu|^2. \quad (102)$$

From now on, we abbreviate the notation $\langle\beta|\cdots|\beta\rangle$ which represent the expectation value in squeezed state to $\langle\cdots\rangle_\beta$ for convenience. The variances of $\hat{q}$ and $\hat{p}$ can be determined by applying these expectation values into Eqs. (37) and (38) (Vogel

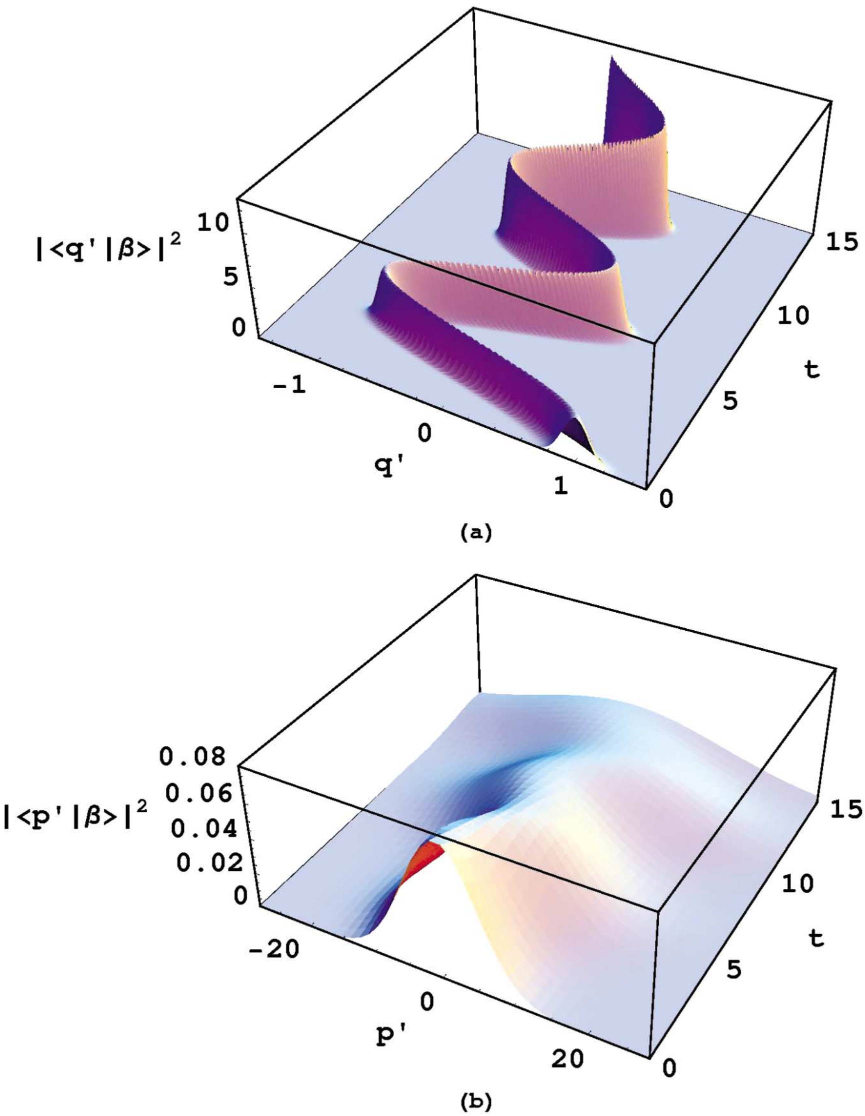
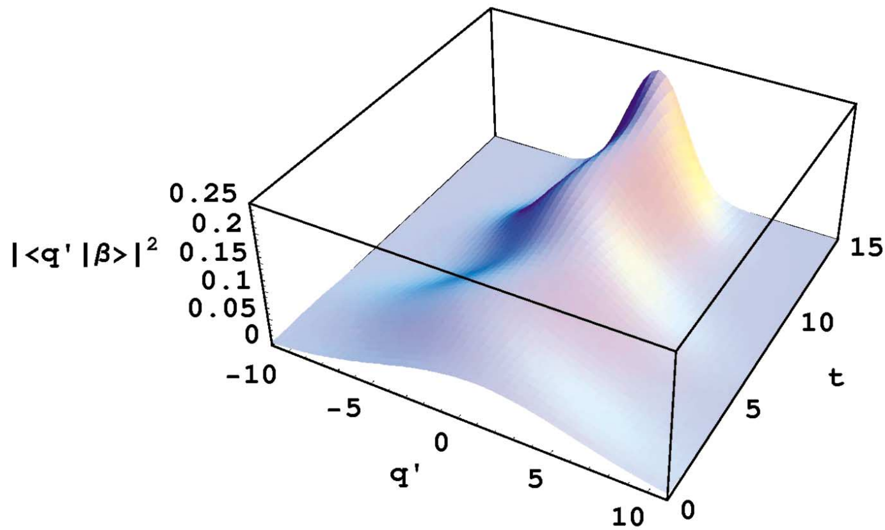
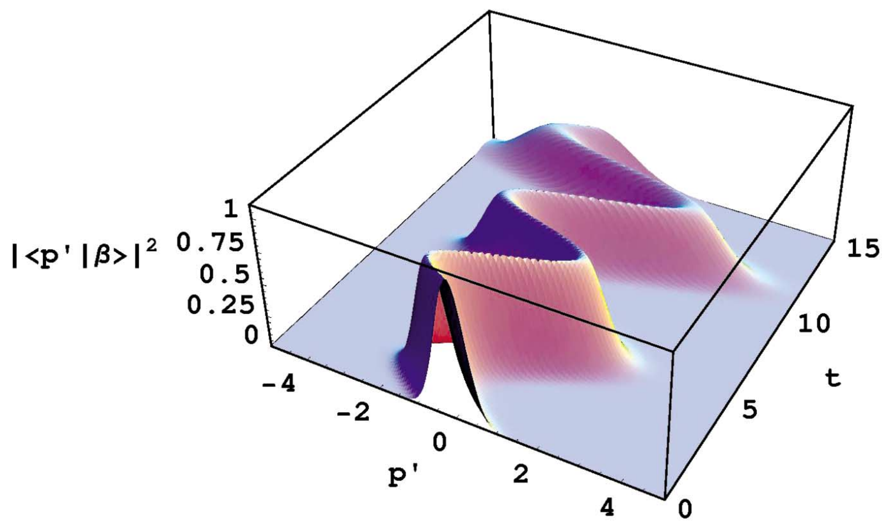


Fig. 3. Probability densities $|\langle q'|\beta\rangle|^2$ (a) and $|\langle p'|\beta\rangle|^2$ (b) in squeezed state as functions of canonical variable and time. We used $\alpha = |\alpha|e^{-i(\Omega t+\phi)}$ with $|\alpha| = 0.7$ and $\phi = 0$, $\hbar = 1$, $\epsilon = 1$, $\sigma = 0.14$, $\omega = 1$, $\lambda = 0$, $|\xi| = 2$, and $\varphi = 0$.



(a)



(b)

Fig. 4. Same as in Fig. 3, but $\varphi = \pi$.

and Welsch, 1994):

$$(\Delta\hat{q})^2_\beta = \frac{\hbar}{2\epsilon\Omega}\exp\left(-\frac{\sigma}{\epsilon}t\right)\left\{1+2|\nu|^2\left[1-\sqrt{1+\frac{1}{|\nu|^2}}\cos(\varphi-2\lambda)\right]\right\}, \tag{103}$$

$$(\Delta\hat{p})^2_\beta = \frac{1}{2}\left(\epsilon\Omega\hbar+\frac{\hbar\sigma^2}{4\Omega\epsilon}\right)\exp\left(\frac{\sigma}{\epsilon}t\right) \times\left\{1+2|\nu|^2\left[1-\sqrt{1+\frac{1}{|\nu|^2}}\cos\left(\varphi-2\lambda+2\phi_p\right)\right]\right\}, \tag{104}$$

where

$$\phi_p = \tan^{-1}\frac{2\Omega\epsilon}{\sigma}+\pi. \tag{105}$$

We depicted the corresponding uncertainty relation in Fig. 5 as a function of conductivity σ. For a certain value of σ the uncertainty relation becomes $\hbar/2$ which is quantum-mechanically acceptable minimum quantity.

On the other hand, the variances of Eqs. (9) and (10) and their commutation relation are

$$(\Delta\hat{X}_1)^2_\beta = \frac{1}{2}\left\{1+2|\nu|^2\left[1-\sqrt{1+\frac{1}{|\nu|^2}}\cos(\varphi-2\lambda)\right]\right\}, \tag{106}$$

$$(\Delta\hat{X}_2)^2_\beta = \frac{1}{2}\left\{1+2|\nu|^2\left[1+\sqrt{1+\frac{1}{|\nu|^2}}\cos(\varphi-2\lambda)\right]\right\}, \tag{107}$$

$$(\Delta\hat{X}_1)^2_\beta(\Delta\hat{X}_2)^2_\beta = \frac{1}{4}[1+4|\mu|^2|\nu|^2\sin^2(\varphi-2\lambda)]. \tag{108}$$

Figure 6 represent the uncertainty relations between $\hat{q}$ and $\hat{p}$ (a) and between $\hat{X}_1$ and $\hat{X}_2$ (b) as functions of magnitude $|\xi|$ and phase φ in Eq. (84). They fluctuate more or less depending on φ.

Performing the similar procedures as the previous case in coherent state and taking advantage of Eqs. (98)–(102), we derive the mean values of the electric and magnetic fields and their squares in squeezed state to be

$$\langle\hat{E}(x,t)\rangle_\beta = \langle\hat{E}(x,t)\rangle_\alpha, \tag{109}$$

$$\langle\hat{E}^2(x,t)\rangle_\beta = \langle\hat{E}^2(x,t)\rangle_\alpha - \frac{\hbar}{2\epsilon V\Omega}\exp\left(-\frac{\sigma}{\epsilon}t\right) \times\left\{\mu\nu\left(\frac{\sigma}{2\epsilon}+i\Omega\right)^2 e^{2ikx}+\mu^*\nu^*\left(\frac{\sigma}{2\epsilon}-i\Omega\right)^2 e^{-2ikx}\right.$$

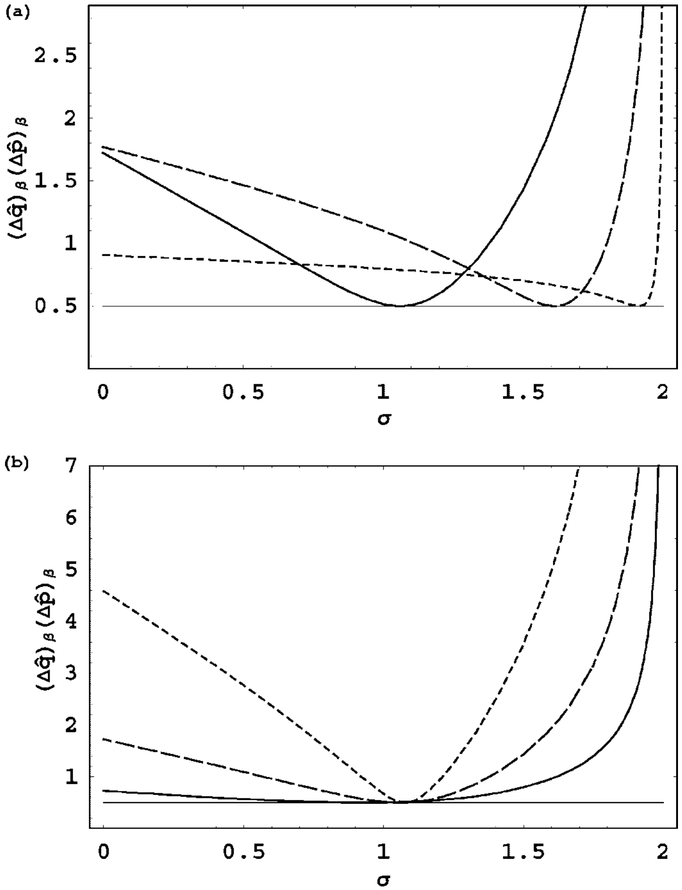


Fig. 5. Uncertainty relation $(\Delta\hat{q})_\beta(\Delta\hat{p})_\beta$ in squeezed state for various values of φ(a) and $|\xi|$(b) as a function of conductivity. The value of φ for (a) is 0(thick dashed line), $\pi/4$(long dotted line), and $\pi/2$(short dotted line). The value of $|\xi|$ for (b) is 0.5(thick dashed line), 1.0(long dotted line), and 1.5(short dotted line). We used $\epsilon = 1$, $\hbar = 1$, $\lambda = 1$, $\omega = 1$, $t = 0$, $|\xi| = 1$ for (a), and $\varphi = 0$ for (b).

$$- 2|\nu|^2\left[\left(\frac{\sigma}{2\epsilon}\right)^2 + \Omega^2\right]\Bigg\}, \tag{110}$$

$$\langle \hat{B}(x,t)\rangle_\beta = \langle \hat{B}(x,t)\rangle_\alpha, \tag{111}$$

$$\langle \hat{B}^2(x,t)\rangle_\beta = \langle \hat{B}^2(x,t)\rangle_\alpha + \frac{\hbar k^2}{2\epsilon V\Omega}\exp\left(-\frac{\sigma}{\epsilon}t\right) \times [\mu\nu e^{2ikx} + \mu^*\nu^* e^{-2ikx} + 2|\nu|^2]. \tag{112}$$

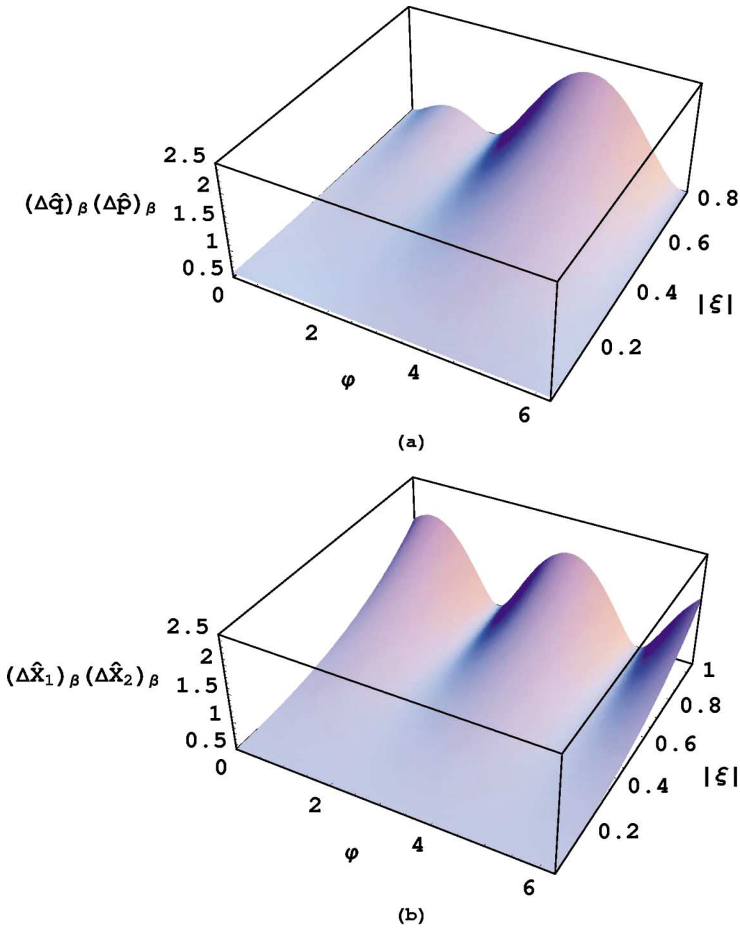


Fig. 6. Uncertainty relation $(\Delta\hat{q})_\beta(\Delta\hat{p})_\beta$(a) and $(\Delta\hat{X}_1)_\beta(\Delta\hat{X}_2)_\beta$(b) in squeezed state as functions of φ and $|\xi|$. We used $\epsilon = 1$, $\hbar = 1$, $\lambda = 1$, $\omega = 1$, $t = 0$, $\sigma = 1$.

Then, we easily identify the dispersion of $\hat{E}$ and $\hat{B}$ field as

$$[\Delta\hat{E}(x,t)]_\beta = \sqrt{\langle\hat{E}^2(x,t)\rangle_\beta - \langle\hat{E}(x,t)\rangle_\beta^2}$$
$$= \sqrt{\frac{\hbar}{2\epsilon V\Omega}}\left[(1+2|\nu|^2)\left(\frac{\sigma^2}{4\epsilon^2}+\Omega^2\right) - \mu\nu\left(\frac{\sigma}{2\epsilon}+i\Omega\right)^2 e^{2ikx}\right.$$
$$\left. -\ \mu^*\nu^*\left(\frac{\sigma}{2\epsilon}-i\Omega\right)^2 e^{-2ikx}\right]^{1/2}\exp\left(-\frac{\sigma}{2\epsilon}t\right), \qquad (113)$$

$$[\Delta\hat{B}(x,t)]_\beta = \sqrt{\langle\hat{B}^2(x,t)\rangle_\beta - \langle\hat{B}(x,t)\rangle_\beta^2}$$
$$= \sqrt{\frac{\hbar}{2\epsilon V\Omega}}\, k \exp\left(-\frac{\sigma}{2\epsilon}t\right)[1 + 2|\nu|^2 + \mu\nu e^{2ikx} + \mu^*\nu^* e^{-2ikx}]^{1/2}. \tag{114}$$

6. SUMMARY

With the choice of Coulomb gauge, we investigated coherent state and squeezed state for the light propagating through homogeneous conducting linear media along the view of the quantization scheme with LR invariant operator method. We evaluated the coherent state by operating $\langle q'|$ and $\langle p'|$ to both side of Eq. (27). The resulting coherent state represented as Eqs. (49) and (51). If we put phases δ_{cq} and δ_{cp} as Eqs. (50) and (52), respectively, these calculations agree with the ones evaluated by expanding the number state eigenfunction $|n\rangle$.

For the case that the media has no charge density, the scalar potential is zero so that the electric and magnetic fields can be represented in terms of only vector potential. By expanding the vector potential, we evaluated quantized electric and magnetic fields. These fields are decayed exponentially, and slowly or rapidly depending on the value of σ/ϵ, with time. Note that the fields suffer continuous dissipation that agrees with the result of Choi (2003a), due to the conductivity of the media. The dissipated field energy become the source of the heat produced to raise the temperature in the media. The probability densities, Eqs. (53) and (54), oscillate back and forth with time about $q' = 0$ and $p' = 0$, respectively. This behavior is very similar to the motion of classical oscillator. The amplitude of oscillation decreases with time in q-space while increases in p-space.

We confirmed that the uncertainty product between the two orthogonal phase amplitudes, $\hat{a}_1$ and $\hat{a}_2$, in coherent state is same as the uncertainty product in vacuum number state. Not only the field strength but also the dispersion of the fields are decayed exponentially with time. From Figs. 1 and 2, we can see that the envelope of the relative noise alternately become large and small with position as well as with time.

We also investigated squeezed states for the dissipative light by introducing a squeeze operator. The squeezed state is an eigenstate of operator $\hat{b}$ defined in Eq. (87). The q- and p-space representations of squeezed state are given by Eqs. (92) and (94). For the light that strongly squeezed in q, the width of the probability density at certain time in q-space is very narrow while that in p-space is much broad. As a matter of course, the phenomenon is vice versa for the light that strongly squeezed in p. The uncertainty relation between canonical variables $\hat{q}$ and $\hat{p}$ are varied depending on the value of conductivity σ in squeezed state, but not lowered below $\hbar/2$ which is quantum-mechanically acceptable minimum uncertainty. We also derived mean values and dispersions of the dissipative electric and magnetic fields in squeezed state.

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